Shear flow of angular grains: acoustic and frictional effects, and stick-slip instabilities

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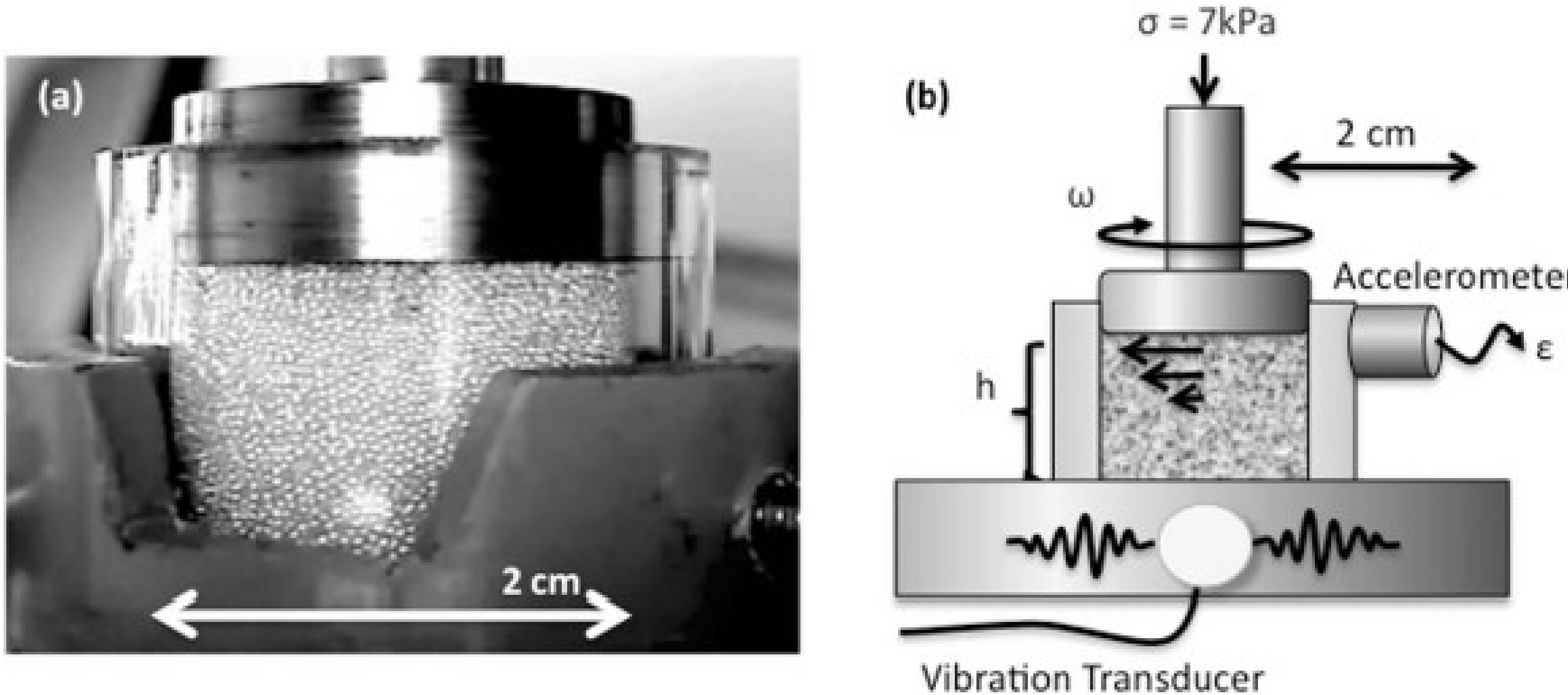
[1] PRE 90, 032204 (2014)
[2] Unpublished results; will be on arXiv later

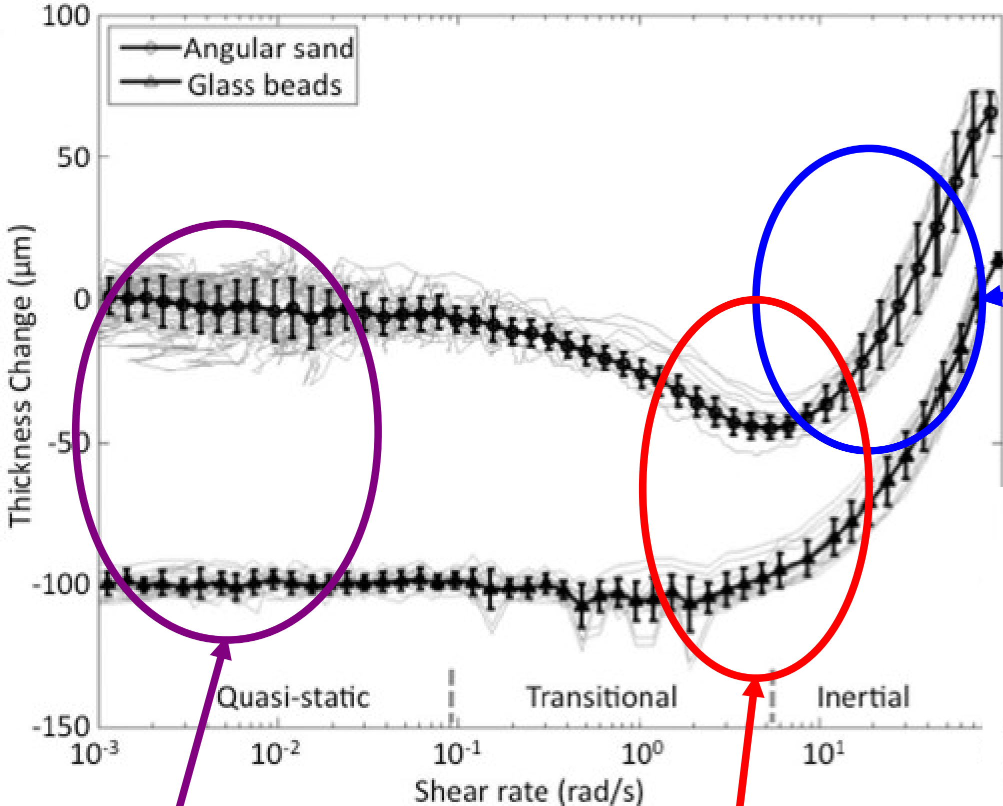
Part I: Variation of shear band thickness or volume

The experiment



N. J. van der Elst, E. E. Brodsky, P.-Y. Le Bas, and P. A. Johnson, J. Geophys. Res. 117, B09314 (2012).

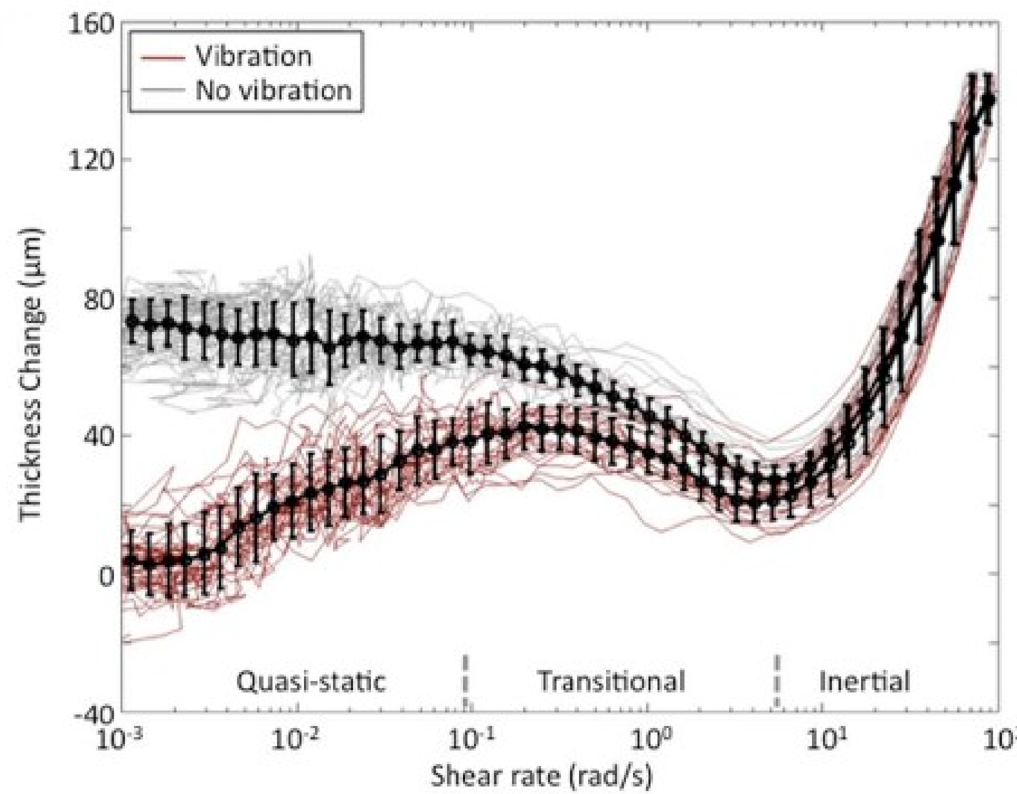
The experiment



Shear-rate dilation

Quasi-static regime

How about this non-monotonic rate behavior?



Statistical thermodynamics

- Assume correspondence between volume and configurational disorder.
- The compactivity

$$X = \frac{\partial V}{\partial S_C}$$

  characterizes the state of configurational disorder of a granular medium, with configurational entropy S_C.
- Extensive volume V varies monotonically with the temperature-like quantity X.

Big picture: Statistical thermodynamics

- The compactivity X increases when external work is done on the system.
- X decreases when entropy flows from the granular medium into the surrounding environment.
- This heat flow is governed by various noise sources within the system, such as those generated by driving forces (shear, tapping) or by friction between particles.

Big picture: Statistical thermodynamics

- First law of thermodynamics:

$$\begin{aligned}\dot{U}_T &= V\,s\,\dot{\gamma}^{\mathrm{pl}} - p\,\dot{V} \\ &= V\,s\,\dot{\gamma}^{\mathrm{pl}} - pX\,\dot{S}_C - p\sum_\alpha \left(\frac{\partial V}{\partial \Lambda_\alpha}\right)_{S_C} \dot{\Lambda}_\alpha\end{aligned}$$

- For simplicity, encapsulate all kinetic-vibrational degrees of freedom in *T*. Then

$$pX\,\dot{S}_C = V\,s\dot{\gamma}^{\mathrm{pl}} - p\sum_\alpha \left(\frac{\partial V}{\partial \Lambda_\alpha}\right)_{S_C} \dot{\Lambda}_\alpha - T\,\dot{S}_T$$

This will be used to derive the solution for *X*.

Big picture: Statistical thermodynamics

- Second law of thermodynamics:

$$\dot{S} = \dot{S}_C + \dot{S}_T \geq 0.$$

- Non-negativity condition constrains microscopic details (internal variables at steady state, etc)
- Rescale variables

$$q \equiv \tau \dot{\gamma}^{\mathrm{pl}},$$

$$\chi \equiv \frac{X}{v_Z}; \quad \theta \equiv \frac{T}{p v_Z}; \quad \mu \equiv \frac{s}{p}$$

Big picture: Statistical thermodynamics

- According to the first law of thermodynamics, the compactivity obeys an equation of the form

$$\epsilon_1 \dot{\chi} = \mu \dot{\gamma}^{\rm pl} - \mathcal{K}(\chi, \theta)(\chi - \theta)$$

- This is the difference between the rate at which plastic work of deformation is done on the system, and the rate at which energy is dissipated through coupling between the configurational and kinetic-vibrational degrees of freedom.
- Presence of other noise sources (tapping ρ, and friction ξ) changes the steady-state behavior.

Big picture: Statistical thermodynamics

- Appeal to hard-sphere limit, in which the only noise source is that of shear Γ.
- One-to-one relationship between shear rate q and compactivity χ (Vogel-Tamann-Fulcher form), describing shear-rate dilatation:

$$\frac{1}{q} = \frac{1}{q_0} \exp\left[\frac{A}{\hat{\chi}} + \alpha_{\rm eff}(\hat{\chi})\right],$$

$$\alpha_{\rm eff}(\hat{\chi}) = \left(\frac{\hat{\chi}_1}{\hat{\chi} - \hat{\chi}_0}\right) \exp\left(-3\frac{\hat{\chi} - \hat{\chi}_0}{\hat{\chi}_A - \hat{\chi}_0}\right).$$

Big picture: Statistical thermodynamics

- Presence of other noise sources (tapping ρ, and friction ξ) changes the steady-state behavior.
- Goal is to understand how, and model the non-monotonic variation of volume.
- The coupling between the configurational and kinetic-vibrational subsystems contains additive contributions from shear, vibrations and friction:

$$\mathcal{K}(\chi, \theta) = \frac{\mathcal{A}}{\tau}(\Gamma + \xi + \rho)$$

Big picture: Statistical thermodynamics

- Now substitute

$$\mathcal{K}(\chi,\theta) = \frac{\mathcal{A}}{\tau}(\Gamma + \xi + \rho)$$

into

$$\epsilon_1 \dot{\chi} = \mu \dot{\gamma}^{\rm pl} - \mathcal{K}(\chi,\theta)(\chi - \theta)$$

- Use the Pechenik relation

$$\Gamma = \tau \mu \dot{\gamma}^{\rm pl} / \epsilon_0 \mu_0 \Lambda$$

(i.e. fluctuations generated by shear proportional to plastic work rate of deformation) and appeal to hard-sphere limit, to compute the coupling constant.

Big picture: Statistical thermodynamics

- Then, in the presence of friction and tapping, the steady-state compactivity is given by

$$\chi^{\mathrm{ss}} = \frac{\Gamma \hat{\chi}(q) + (\xi + \rho)\theta}{\Gamma + (\xi + \rho)}$$

shearing friction tapping

which has a one-to-one correspondence to volume V.

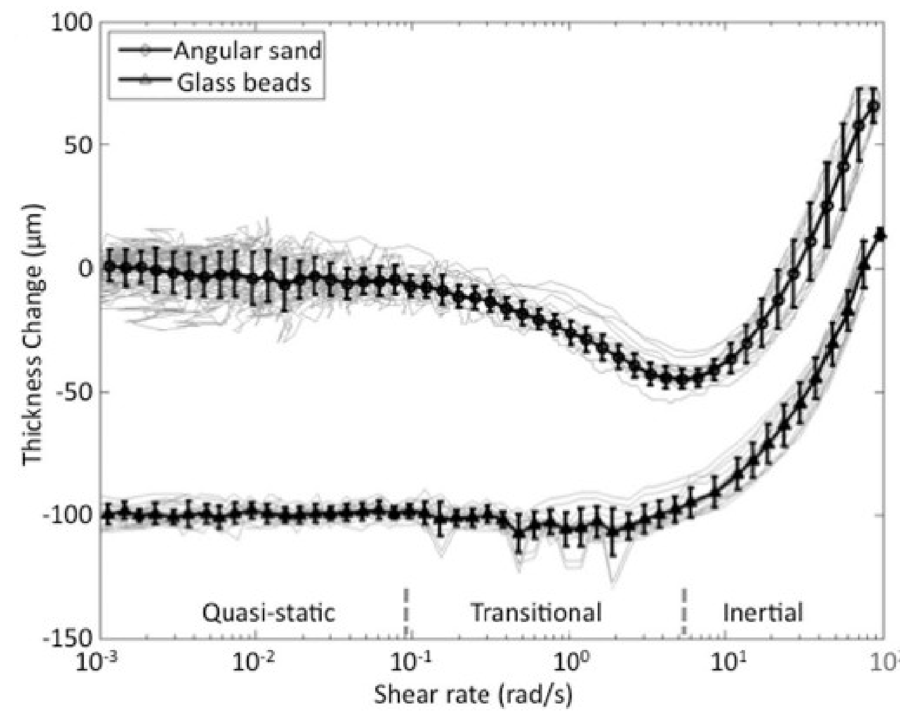
- Competition between shear-induced dilation and noise-induced compaction

Big picture: Frictional noise

- Consider no-vibration case ($\rho = 0$) for the time being.

$$\chi^{\rm ss} = \frac{\Gamma\hat{\chi}(q) + (\xi+\rho)\theta}{\Gamma + (\xi+\rho)}$$

- Mechanical noise $\Gamma \propto \dot{\gamma}^{\rm pl}$.
- If "frictional noise" is quadratic in the shear rate when it is small, and saturates at large shear rates, then $\hat{\chi}(q)$ dominates in both limits → non-monotonicity!



$$\xi = \xi_0 \tanh[(\tau_f \dot{\gamma}^{\rm pl})^2]$$

Microscopic model

- Statistical thermodynamics is sufficient to account for qualitative behavior.
- The rest is microscopic detail that quantitatively connects the volume V, the compactivity χ, and the shear rate q.

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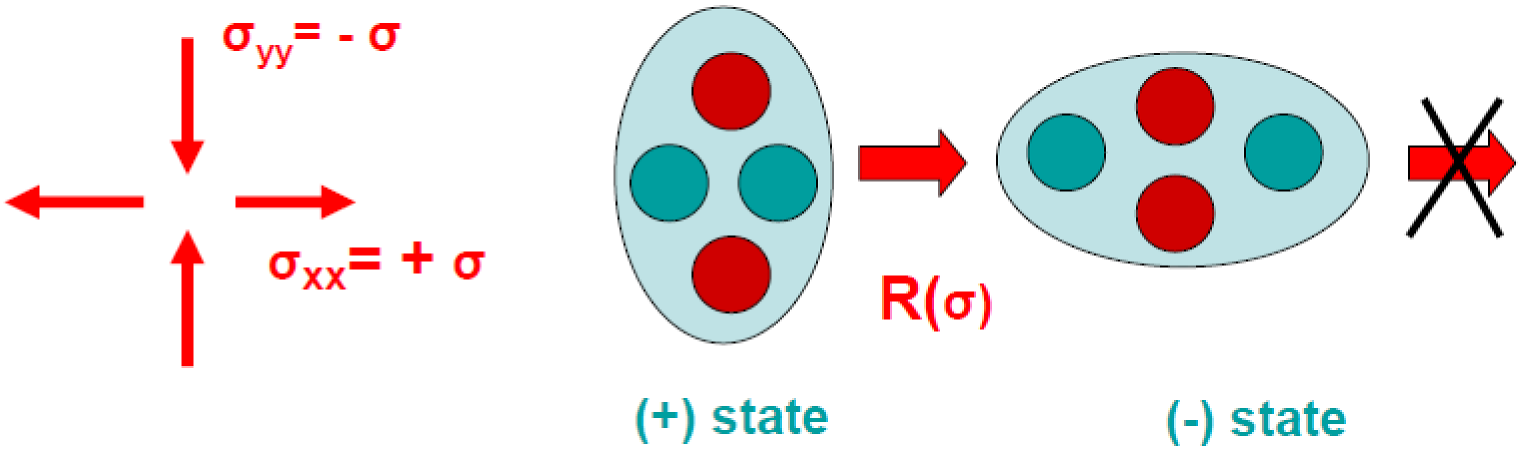
Microscopic model

- Combination of shear transformation zones (STZ's) and misalignment defects



- STZ transitions (particle rearrangements):

$$\tau \dot{N}_{\pm} = \mathcal{R}(\pm s) N_{\mp} - \mathcal{R}(\mp s) N_{\pm} + \tilde{\Gamma} \left(\frac{1}{2} N^{eq} - N_{\pm} \right)$$

Microscopic model

- Plastic strain rate accounted for by granular rearrangements:

$$\dot{\gamma}^{\rm pl} = \frac{2v_0}{\tau V}\left(\mathcal{R}(s)N_- - \mathcal{R}(-s)N_+\right)$$

- Using second law of thermodynamics, the density of STZ's $\Lambda \equiv (N_+ + N_-)/N$ at steady state is given by

$$\Lambda^{\rm eq} = 2\exp(-1/\chi).$$

Microscopic model

- For angular grains, misalignments between nearest neighbors creates excess volume.
- Suppose each grain is assigned an orientation + and – , and define the orientational bias

$$\eta \equiv \frac{N_+^G - N_-^G}{N}.$$

- Ising-like interaction (cf. J. S. Langer, PRE 88, 012122, 2013): volume gains an extra term $\sim -\eta^2$.

Microscopic model

- Orientation changes as a result of noise
- Postulate master equation

$$\tau \dot{N}_{\pm}^{G} = \mathcal{R}_{\pm}^{G} N_{\mp}^{G} - \mathcal{R}_{\mp}^{G} N_{\mp}^{G}.$$

  in analogy to STZ transitions.
- Need a relationship between volume V and orientational bias η to derive steady-state orientational bias using second law of thermodyanmics.

Microscopic model

- $V(\chi)$ crucial for theory validation.
- Assume that aside from the η-dependent term, the effective volume expansion coefficient is linear:

$$V = V_0 + (\hat{\chi} - \hat{\chi}_0)V_1 - N\eta^2 v_a.$$

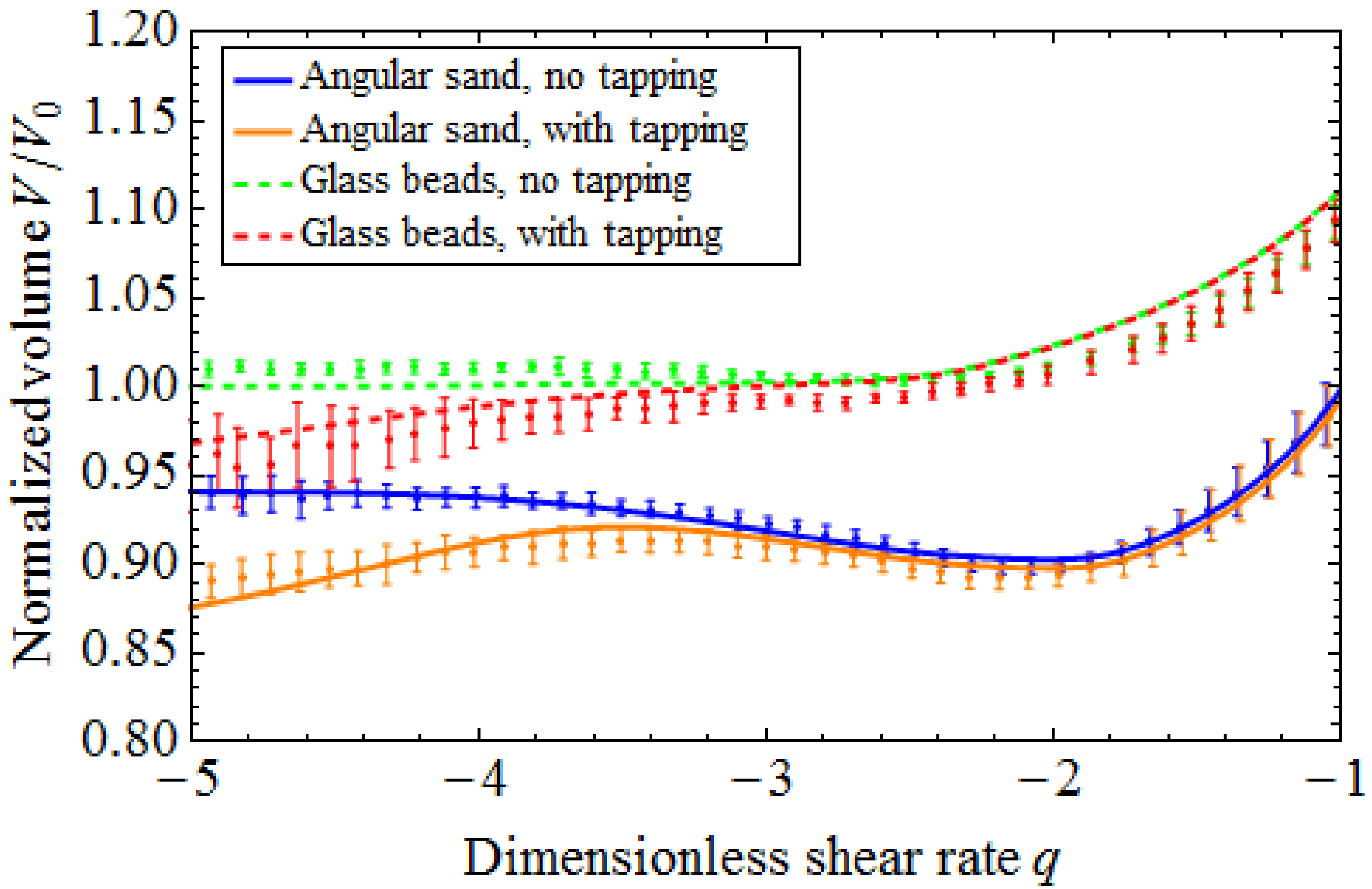
- At steady state. the orientational bias obeys

$$\eta^{\mathrm{eq}} = \tanh\left(\frac{2\eta^{\mathrm{eq}} v_a}{\chi}\right)$$

and is governed by the same compactivity χ that controls the STZ density (more on this point later).

Quantitative agreement with experiment

- Volume as a function of shear rate (this fit is better than the one in the paper where we used a somewhat different interpolation for ξ)



What we have learned so far

- Friction can be interpreted as a kind of "noise".
- Frictional "noise" couples configurational and kinetic-vibrational subsystems.
- Effect on volume amplified by angularity which introduces misalignment defects.
- Tapping results in shear strength reduction.
- Implication on acoustic fluidization of earthquake faults, and dynamic triggering of earthquakes?

What we have learned so far

- Non-monotonic variation of system volume with shear rate is purely a consequence of effective-temperature thermodynamics and coupling between different degrees of freedom, independent of microscopic model
- However, model of misalignments is physically motivated, and fits well with data.

Some remarks

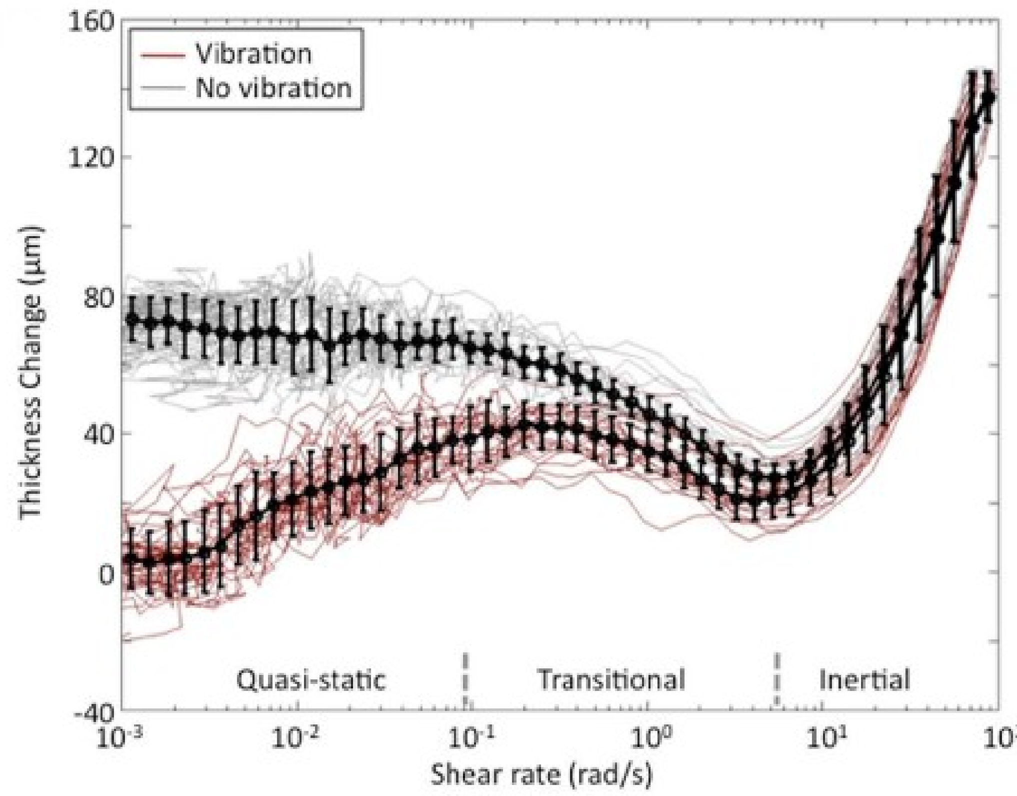
- Central assumption: misalignment defects and STZ's governed by the same "disorder temperature" χ.
- Justification: inverse of tapping frequency (~ 40.2 kHz) is similar to inertial time scale

$$\tau = a\sqrt{\rho_G/p}$$

  at a pressure p = 7 kPa.
- What if this had not been the case?

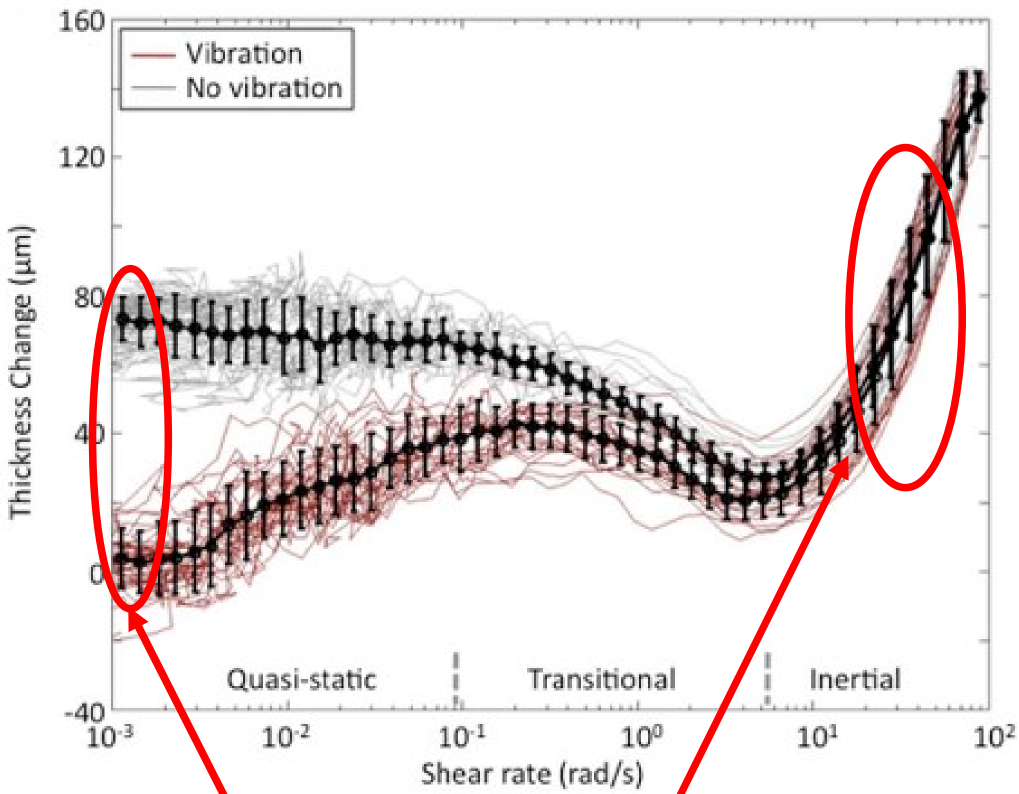
Some remarks

- What if the inverse tapping frequency and the inertial time scale are several orders of magnitude apart?
- This may be achieved by varying the pressure, grain size or tapping frequency.



Some remarks

- Will STZ's and misalignments be governed by two different χ's?
- This provides a stringent test on nonequilibrium thermodynamics.



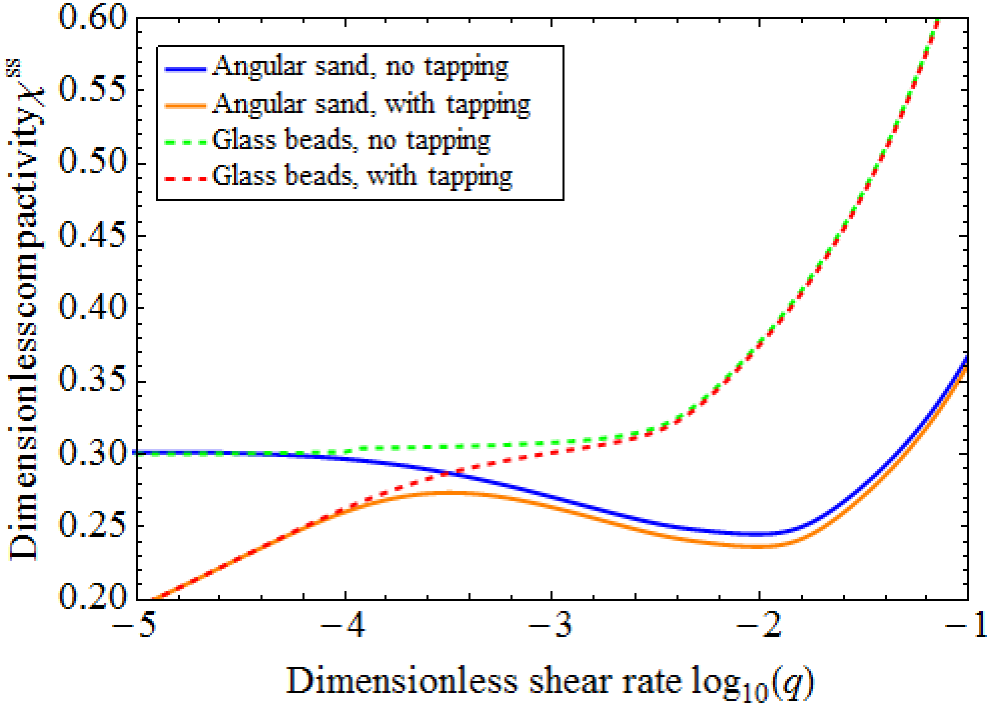
$$\chi^{ss} = \frac{\Gamma\hat{\chi}(q) + (\xi + \rho)\theta}{\Gamma + (\xi + \rho)}$$

May not be true anymore!

Part II: Relating interparticle friction and nonmonotonic rheology to stick-slip instabilities

Non-monotonicity implies instability?

- Observe that interparticle friction brings the steady-state disorder temperature χ down to below χ_0, a "forbidden" region for hard spheres.
- If the minimum flow stress varies with χ in that region, then there can be a rate-weakening regime.



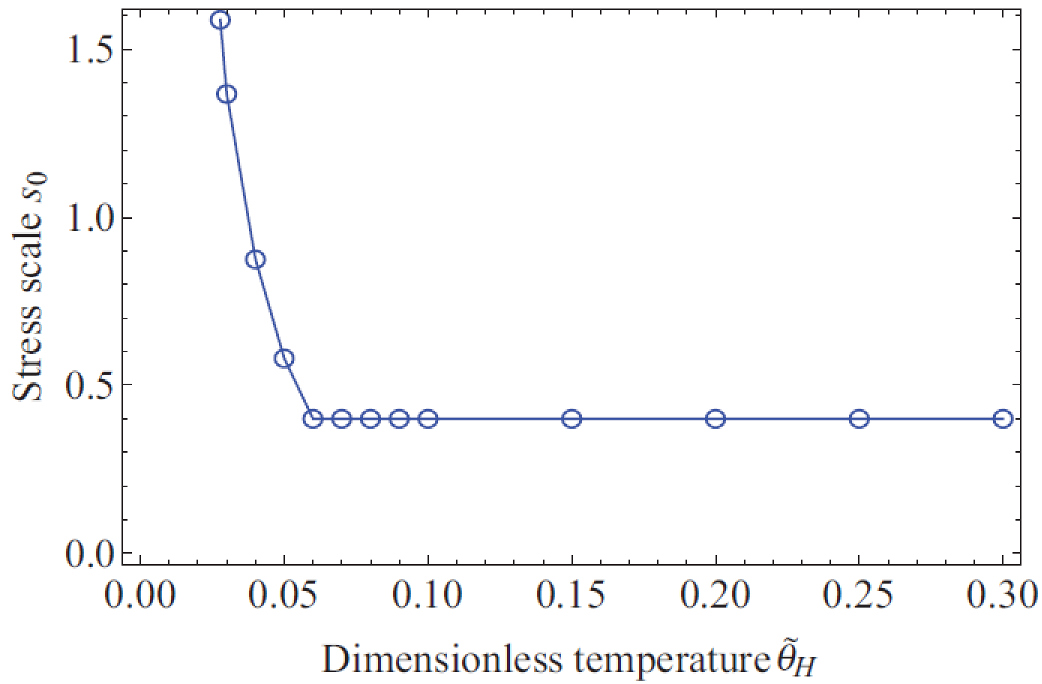
Non-monotonicity implies instability?

- The parameter μ_0 that controls the minimum flow stress originates from the Pechenik relation:

$$\Gamma = \tau\mu\dot{\gamma}^{\rm pl}/\epsilon_0\mu_0\Lambda$$

- Intuitively, if the granular medium is more tightly packed, less fluctuations can be generated by shear, so that μ_0 should be a decreasing function of χ.
- In our analysis of hard-sphere simulations [PRE 85, 061308 (2012)], we "measured" μ_0 and found that it decreases with temperature in the low-temperature regime, below the glass transition:

Non-monotonicity implies instability?

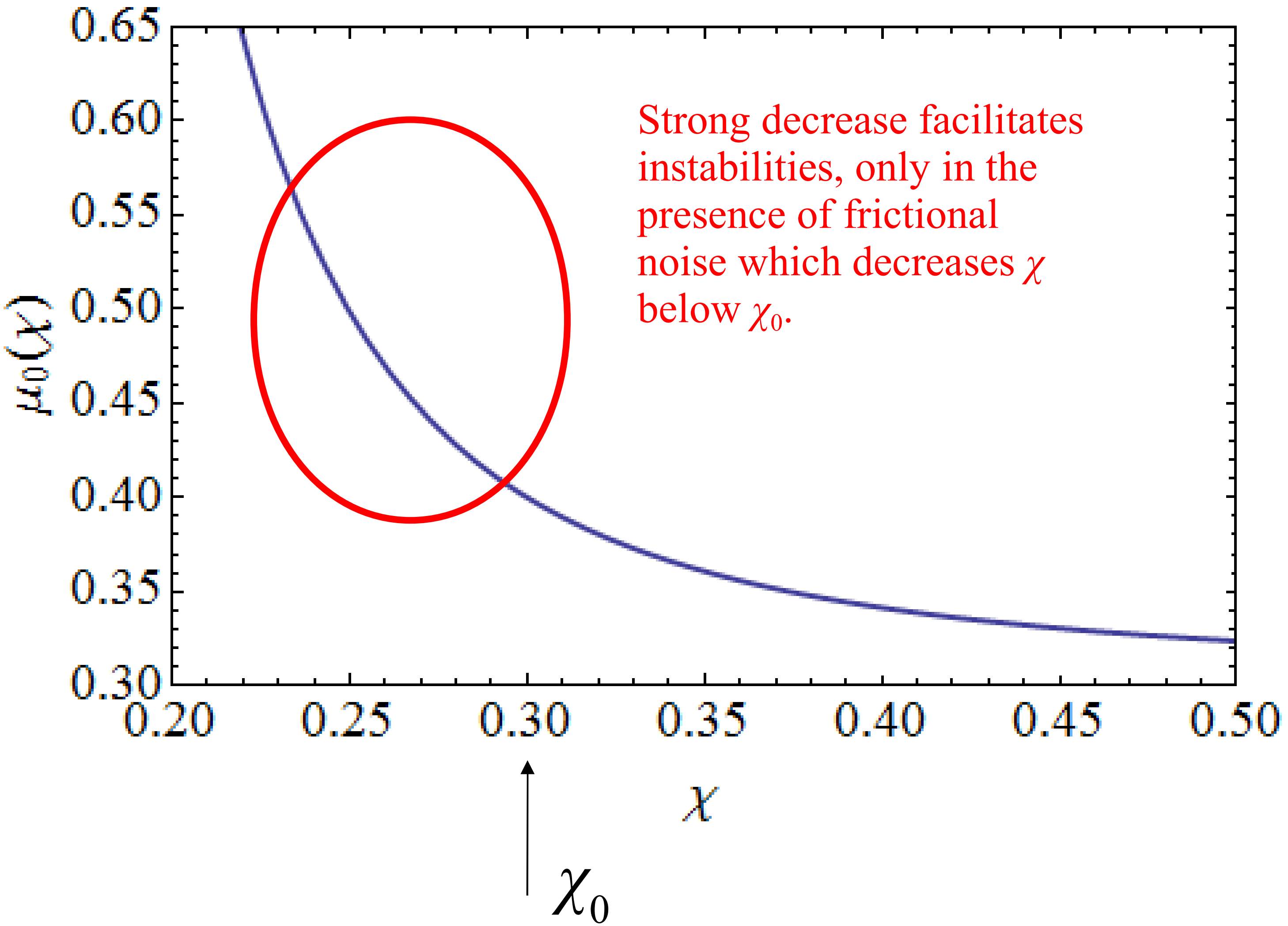


[CKCL and JSL, PRE 85, 061308 (2012)]

- It is equally plausible that μ_0 should be a decreasing function of χ.

Non-monotonicity implies instability?

- We pick a μ_0 that looks like this:



Non-monotonicity implies instability?

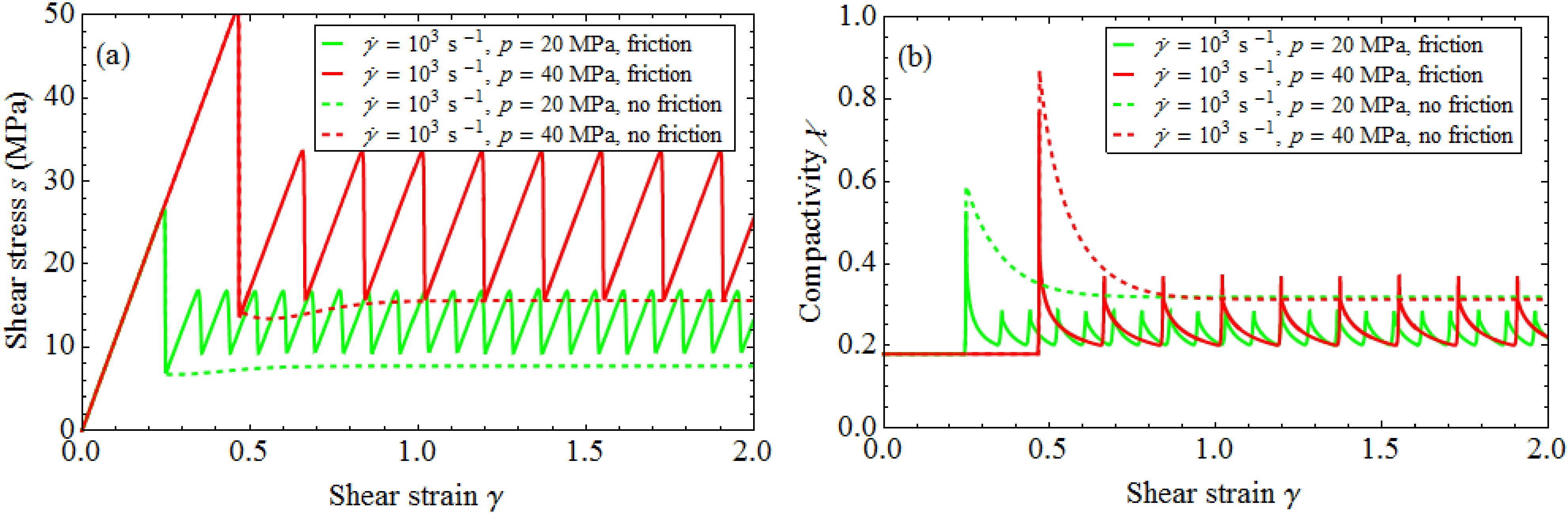
- Dynamical equations:

$$\dot{\mu} = (G/p)(\dot{\gamma} - \dot{\gamma}^{\mathrm{pl}}),$$

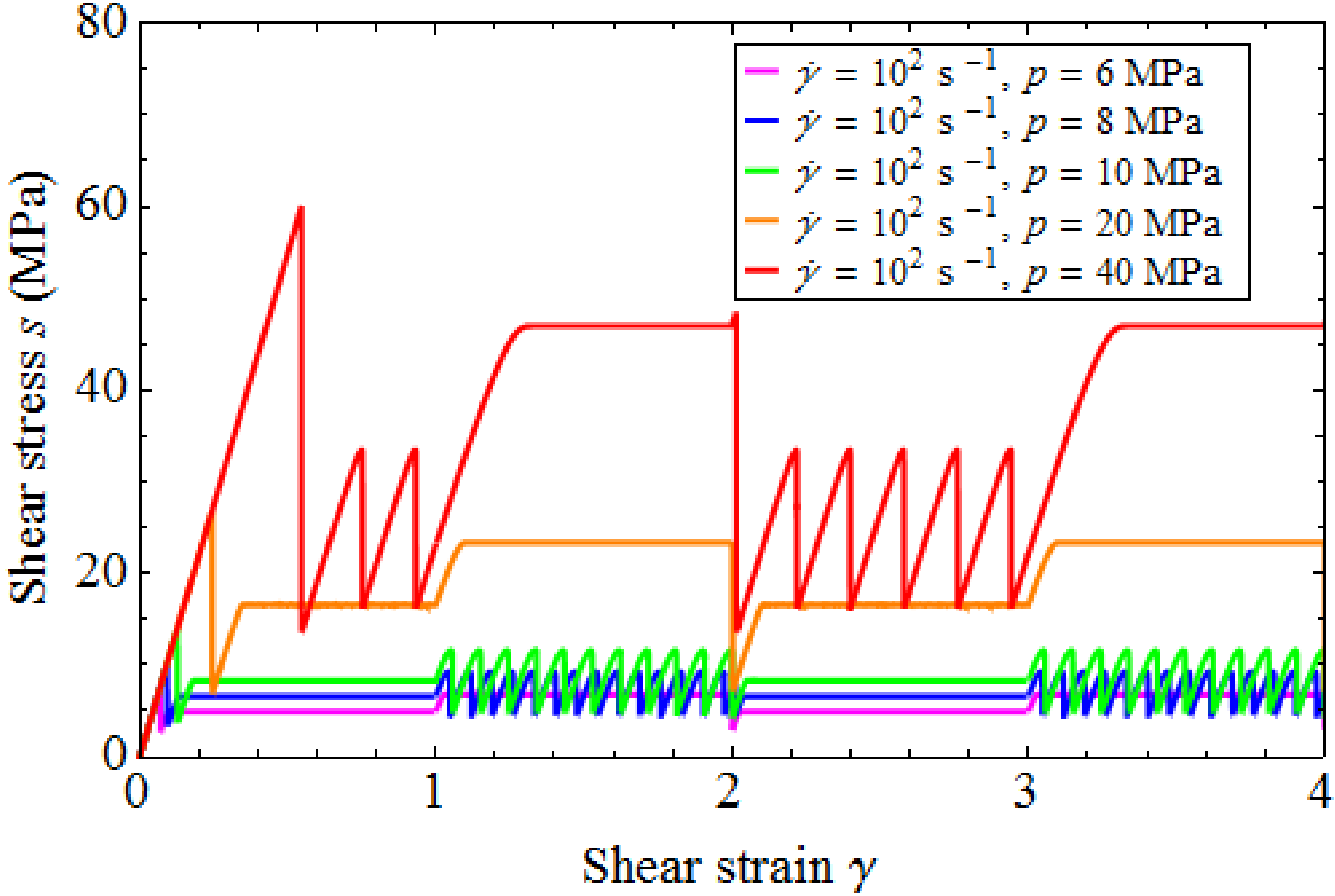
$$\dot{\chi} = \frac{\epsilon_0 \mu_0}{\tau \epsilon_1} \frac{2e^{-1/\chi}}{\hat{\chi}(q) - \theta} \left[\Gamma(\hat{\chi}(q) - \chi) + (\xi + \rho)(\theta - \chi) \right].$$

- The equation for χ is a consequence of the first law of thermodynamics.
- Misalignments not important as they are not directly coupled to shear deformation.
- We show below that interparticle friction causes stick-slip instabilities.

Theoretical predictions



Theoretical predictions



not vibrated | vibrated | not vibrated | vibrated

Concluding remarks

- Interparticle friction "cools" down sheared granular flow by reducing the volume.
- Under generic assumptions on the fluctuations produced by shearing (noise strength dependent on disorder or "looseness"), this cooling is a source of instability.
- Vibrations unjam the granular medium, and can either promote or suppress stick-slip depending on the pressure and shear rate.

For full details, please refer to
PRE 90, 032204 (2014).

The stick-slip analysis will be available on arXiv shortly.

Thank you!